Tabla de bifurcaciones básicas en una dimensión

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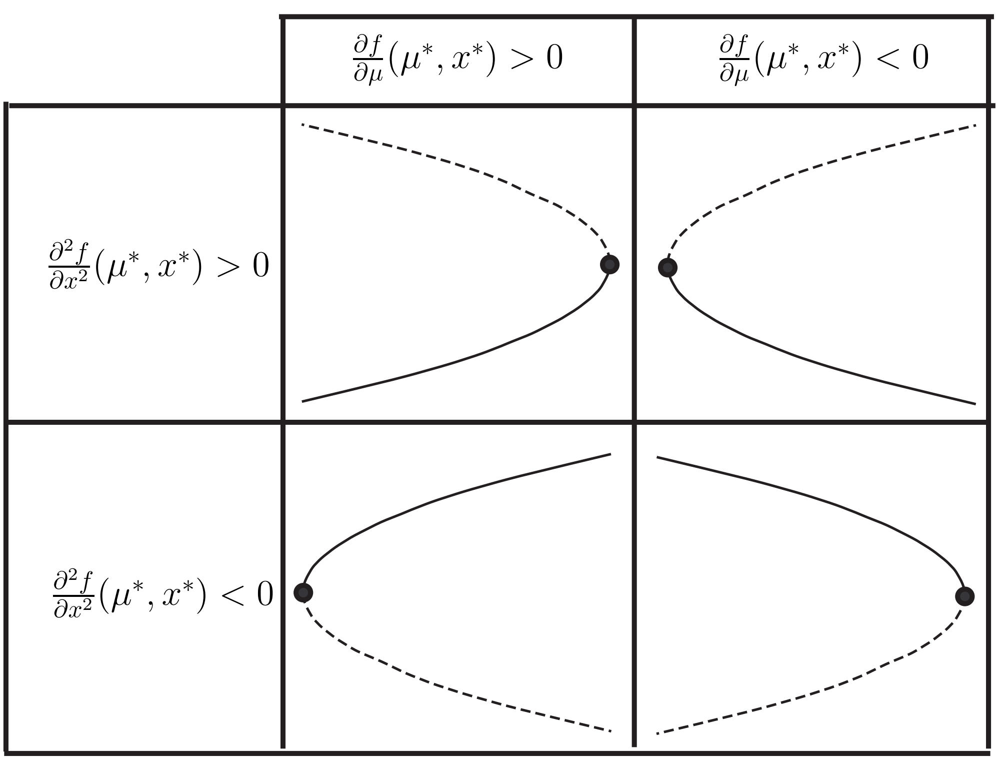
Consideremos un sistema dinámico de una dimensión de la forma $\dot{x} = f(\mu, x)$, donde el dominio de f es un abierto de $\mathbb{R}^2$. Se dirá que (μ^*, x^*) es un punto de bifurcación si

$$f(\mu^*, x^*) = 0, \qquad \frac{\partial f}{\partial x}(\mu^*, x^*) = 0.$$

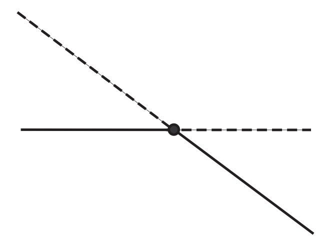
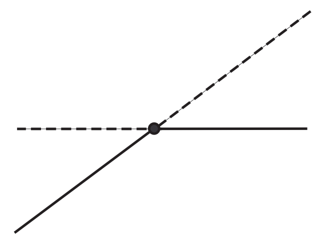
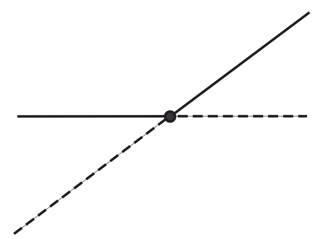
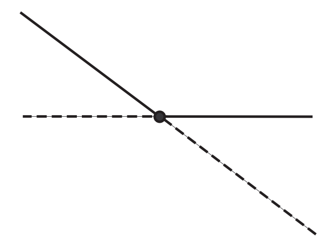
Sea $\Delta = f_{xx}(\mu^*, x^*) f_{\mu\mu}(\mu^*, x^*) - f_{x\mu}(\mu^*, x^*)^2$. Dado un punto de bifurcación, éste se clasifica según la siguiente tabla.

Bifurcación	$\frac{\partial f}{\partial \mu}(\mu^*, x^*)$	$\frac{\partial^2 f}{\partial x^2}(\mu^*, x^*)$	$\frac{\partial^2 f}{\partial x \partial \mu}(\mu^*, x^*)$	$\frac{\partial^3 f}{\partial x^3}(\mu^*, x^*)$	Δ	Forma Normal
Nodo-silla	$\neq 0$	$\neq 0$				$\pm\mu \pm x^2$
Transcrítica	0	$\neq 0$	$\neq 0$		< 0	$\pm\mu x \pm x^2$
Tenedor	0	0	$\neq 0$	$\neq 0$	< 0	$\pm\mu x \pm x^3$

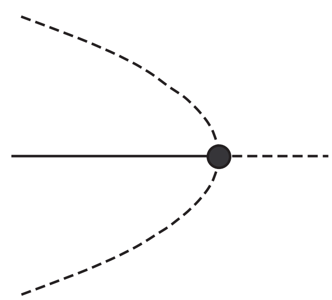
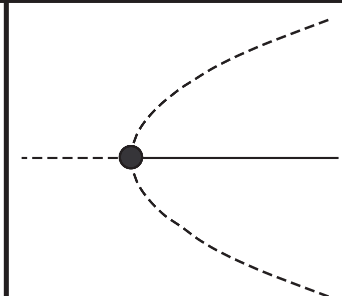
1. Bifurcación Nodo-silla



2. Bifurcación Transcrítica

	$\frac{\partial^2 f}{\partial x \partial \mu}(\mu^*, x^*) > 0$	$\frac{\partial^2 f}{\partial x \partial \mu}(\mu^*, x^*) < 0$
$\frac{\partial^2 f}{\partial x^2}(\mu^*, x^*) > 0$		
$\frac{\partial^2 f}{\partial x^2}(\mu^*, x^*) < 0$		

3. Bifurcación Tenedor

	$\frac{\partial^2 f}{\partial x \partial \mu}(\mu^*, x^*) > 0$	$\frac{\partial^2 f}{\partial x \partial \mu}(\mu^*, x^*) < 0$
$\frac{\partial^3 f}{\partial x^3}(\mu^*, x^*) > 0$		
$\frac{\partial^3 f}{\partial x^3}(\mu^*, x^*) < 0$